COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on ''Model kinetic equation for low-density granular flow''

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Hydrodynamic equations for a system of inelastically colliding granules were systematically derived on the basis of the Boltzmann equation by the Chapman-Enskog method. [A. I. Goldshtein, V. N. Poturaev, and I. A. Shulyak, Fluid Dyn. (USSR) **25**, 305 (1990); A. Goldshtein and M. Shapiro, J. Fluid Mech. **282**, 75 (1995)]. We feel that this problem has recently been incorrectly treated by J. J. Brey, F. Moreno, and James W. Dufty [Phys. Rev. E **54**, 445 (1996)] using the Bhatnagar-Gross-Krook approximation of the kinetic equation. In this Comment, the inconsistency of their approach is analyzed, and possible adequate methods for stability analyses of the homogeneous state of inelastic granules are delineated. [S1063-651X(98)10804-8]

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In a recent paper [1] Brey, Moreno, and Duffy used a variant of Bhatnagar-Gross-Krook (BGK) kinetic model equation to investigate the time evolution of a low-density granular gas composed of inelastically colliding particles. The kinetic equation was solved by a modification of the Chapman-Enskog method by expanding about a local homogeneous state in powers of gradients of the hydrodynamic properties. The velocity distribution function describing the hydrodynamics of the homogeneous state was found to decay as a power function for large velocities, which is in contrast with the Maxwellian distribution prevailing for molecular fluids. As a consequence of this algebraic decay, the velocity moments of degree $k \geqslant 2/(1-e^2)$, with e being the restitution coefficient, diverge. Furthermore, the energy balance equation was found to contain, in addition to a sink term describing the energy loss in collisions, a contribution to the heat flux, which is proportional to the gradient of the granular gas number density.

Part of the above results were obtained earlier [2,3] for a more elaborate collisional model, a wider range of granular gas density and a more general kinetic equation of the Boltzmann-Enskog type. This equation, derived for a system of inelastic rough spheres, was treated by the Chapman-Enskog method to derive the pertinent hydrodynamic equations. As a result, the sink terms were calculated and the additional terms contributing to the heat flux were revealed, which are proportional to grad ln n and grad ln ΔP, with n and ΔP being the particle number density and gas pressure correction due to collisional momentum transfer in dense gases, respectively. We also showed [2,3] that a dense granular gas composed of rough particles is characterized by an additional sink and energy loss terms arising from the gas compression (or expansion).

In particular, our analyses of the homogeneous hydrodynamic state of inelastic smooth granules, performed on the basis of the Boltzmann-Enskog kinetic equation, showed [3] that the granular velocity distribution function is *close to the Maxwellian*. This basically opposes the results obtained by Brey, Moreno, and Duffy [1]. The first nontrivial coefficient in the expansion of the velocity distribution function in series of Sonine polynomials is small [<0.04 (see Ref. [3])] for all e. This coefficient produces the fourth order velocity moment, which does not exist according to the study of Ref. [1] for $e^2<\frac{1}{2}$. The latter contradiction is clearly attributed to the apparent deficiency of their variant of the BGK equation. It contains the two unknown quantities: collisional frequency ζ and a function f_0, determining a reference state. These quantities appear in the linearized collisional term, and no rationale exist for choosing them during the solution.

For conservative (molecular) gases choice of the collisional term in the BGK kinetic equation is dictated by the following principles: (i) molecules tend to reach the equilibrium state, which is characterized by the Maxwell-Boltzmann distribution [4], and (ii) the information entropy tends to a maximum value [4]. The above principles, used in Ref. [1] are generally not valid for a gas composed of inelastic ($e<1$) particles due to the nonconservative nature of such granular systems. (i) There is no equilibrium state for granular systems; that is, the distribution function always remains t-dependent and so does f_0. Even in the hydrodynamic stage of the system evolution f_0 may be far from the Maxwell-Boltzmann distribution. It may be taken from other studies (e.g., numerical simulations) but it cannot be deter-

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mined in the framework of the version of BGK method chosen. (ii) It is not clear whether the principle of maximum information entropy can be used for systems of inelastically colliding granules. At any rate, this is a matter of a separate investigation, which the authors did not perform.

Given all the difficulties and shortcomings of the BGK methods in application to granular systems, reliable results can be obtained only on the basis of a more elaborate kinetic model which is free from the above *ad hoc* hypotheses. This is the Boltzmann equation, which in fact serves an underlaying basis of BGK models for conservative gases. For a particular case of a dilute granular gas of inelastic smooth spheres this equation may be written in the form [5]:

$$\frac{\partial f(\mathbf{v}_1,t)}{\partial t}=\sigma^2 J(f,f,\mathbf{v}_1,e), \tag{1}$$

$$J(f,f,\mathbf{v}_1,e)=\int d^3\nu_2 d^2k(\mathbf{k}\cdot\mathbf{v}_1)\Theta(\mathbf{k}\cdot\mathbf{v}_{21})\times\left[\frac{1}{e^2}f(\mathbf{v}_1'',t)f(\mathbf{v}_2'',t)-f(\mathbf{v}_1,t)f(\mathbf{v}_2,t)\right], \tag{2}$$

where σ is the sphere diameter, $f(\mathbf{v}_1,t)$ is the velocity distribution function, θ is the Heaviside function, $\mathbf{k}$ is a unit vector pointing from the center of the sphere 2 to the center of the sphere 1, and $\mathbf{v}_{21}\equiv\mathbf{v}_2-\mathbf{v}_1$. Here double printed values denote precollisional velocities of the two spheres [6]:

$$\mathbf{v}_1''=\mathbf{v}_1+\frac{1+e}{2e}(\mathbf{v}_{21}\cdot\mathbf{k})\mathbf{k},\mathbf{v}_2''=\mathbf{v}_2-\frac{1+e}{2e}(\mathbf{v}_{21}\cdot\mathbf{k})\mathbf{k}. \tag{3}$$

According to the main idea of the Champan-Enskog method, the distribution function may depend on time only through five hydrodynamic values, i.e., the particle number density n, the average velocity $\mathbf{u}$, and the kinetic energy E. The solution of the homogeneous problems (1)–(3) may be found in the form [2]

$$f(\mathbf{v}_i,t)=\frac{n}{(E/m)^{3/2}}F_i,\quad F_i\equiv F(V_i^2,e),$$

$$\mathbf{V}_i\equiv(\mathbf{v}_i-\mathbf{u})/\sqrt{E/m},\quad i=1,2 \tag{4}$$

where m is the particle mass. Normalization conditions for function F are

$$\int d^3V_1F_1=\tfrac{1}{2}\int d^3V_1V_1^2F_1=1. \tag{5}$$

Since n and $\mathbf{u}$ are constants, the continuity and momentum equations are identically satisfied. Equations (1)–(4) yield an equation for the evolution of the granular kinetic energy E,

$$\frac{dE}{dt}=n\sigma^2E^{3/2}K(F,F)\equiv-n\sigma^2E^{3/2}\frac{\pi(1-e^2)}{16}\int d^3V_1d^3V_2F_1F_2V_{21}^3, \tag{6}$$

wherein F is the solution of the following integrodifferential equation

$$-K(F,F)\left(\frac{3}{2}F_1+V_1^2\frac{dF_1}{dV_1^2}\right)=J(F,F,V_1,e), \tag{7}$$

and $K(F,F)$ is defined in Eq. (6).

Equations (6), (7), and (2) constitute the main result of the hydrodynamic model applied to a spatially homogeneous system. The system of equations (6), (7), and (2) is closed (does not require a choice of any parameters or functions), free from *ad hoc* hypotheses and much simpler than the Boltzmann equation (1). Equation (6) describing the kinetic energy evolution predicts the power law long time decay of E in the spatially homogeneous case, [6] i.e., $E\propto t^{-2}$. Full solution for E requires determination of $K(F,F)$ from Eq. (7), which may be done by different methods. In particular we solved this problem by expanding function F in series of Sonine polynomials [3], and showed that F is close to Maxwellian for all e at least for moderate values of V. More recently, the latter result was confirmed numerically [7]. Esipov and Poeschel [7] showed that, for large V, the function F decays exponentially.

In conclusion, we will note that by using a less accurate but simpler model, Brey, Moreno, and Duffy [1] were able to recover the existence of additional components in the kinetic-energy flux, stemming from the density gradients and nonconservative nature of the system in agreement with the results in our papers [2,3]. These components are relevant in studying the granular system stability. Another issue that may be relevant and not treated in Ref. [1] is the rate of approach of the solution of the Boltzmann-Enskog equation to the hydrodynamic solution. We have shown [3] that this time rate is not exponentially fast, but has a slower power law t dependence. For e sufficiently close to unity, this power is shown to be negative. In a more general case of arbitrary $0<e<1$, this may not be true and needs a separate investigation.

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